
Particle physics models of inflation

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Summary. Inflation models are compared with observation on the assumption that the curvature perturbation is generated from the vacuum fluctuation of the inflaton field. The focus is on single-field models with canonical kinetic terms, classified as small- medium- and large-field according to the variation of the inflaton field while cosmological scales leave the horizon. Small-field models are constructed according to the usual paradigm for beyond Standard Model physics.¹

1 Introduction

Several different types of inflation model have been proposed over the years. In this survey they are compared with observation on the assumption that the curvature perturbation is generated during inflation. The survey is based on works with my collaborators, in particular [1, 2, 3, 4].

I focus largely on the slow-roll paradigm, because it is the simplest and most widely-considered possibility. It assumes that the energy density and pressure dominated by the scalar field potential V , whose value hardly varies during one Hubble time. Unless otherwise stated, we consider single-field inflation, where just one canonically-normalized ‘inflaton’ field ϕ has significant time-dependence.

In the vacuum, $V = 0$. To generate the inflationary value of V , one or more fields must be strongly displaced from the vacuum and there are two simple possibilities. In non-hybrid inflation, V is generated almost entirely by the displacement of the inflaton field from its vacuum, while in hybrid models it is generated almost entirely by the displacement of some other field χ , called the waterfall field because its eventual descent to the vacuum is supposed to be very rapid. Hybrid models are not at all artificial, being based on the concept of spontaneous symmetry breaking and restoration which is ubiquitous in early-universe cosmology.

The first slow-roll model, termed New Inflation [5] (see also [6]), was non-hybrid. It made contact with particle physics through the use of a GUT theory, but was quickly seen to generate too big a curvature perturbation [7]. Viable models using

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a GUT and supersymmetry were developed, including [8] what were later called hybrid inflation models. The models were rather complicated, in part because of a demand that the initial condition for observable inflation is to be set by an era of thermal equilibrium.

It was gradually recognized that prior thermal equilibrium is not necessary. A second strand of model-building, characterized by little contact with particle physics and focusing exclusively on non-hybrid models, began with the proposal of chaotic inflation [9]. Considerable attention was paid to non-Einstein gravity theories, notably the proposal of Extended Inflation [10]. In its original form that proposal is not viable if the inflaton perturbation generates the curvature perturbation [11], though it becomes viable if the curvature perturbation is generated afterward [12].

Following the formulation of a simple hybrid inflation model [14], attention went back to the connection with particle physics and supersymmetry. Almost all proposals for field theory beyond the Standard Model were considered as arenas for inflation model-building, including especially GUTs and the origin of low-energy supersymmetry breaking.

The most recent phase of model-building, beginning in about 2000, is based directly on brane world scenarios. We will consider the prediction of these kind of models without describing their string-theoretic derivation.

2 Beyond the Standard Model

We begin with some general ideas about the very early Universe, taking on board current thinking about what may lie beyond the Standard Model of particle physics. Guided by the desire to generate primordial perturbations from the vacuum fluctuation of scalar fields, one usually supposes that an effective four-dimensional (4-d) field theory applies after the observable Universe leaves the horizon, though not necessarily with Einstein gravity.

To generate perturbations from the vacuum fluctuation we need $|aH|$ to increase with time, which is achieved by inflation defined as an era of expansion with $\ddot{a} > 0$ (repulsive gravity).² Perturbations would also be generated from the vacuum during an era of contraction with $\ddot{a} < 0$. The original suggestion was called the pre-Big-Bang [15]. A more recent version where the bounce corresponds to the collision of branes was called the ekpyrotic Universe [16], which was further developed to produce a cyclic Universe [17]. In these scenarios, the prediction for the perturbation depends crucially on what happens at the bounce, which is presently unclear.

Returning to the inflationary scenario, the 4-d field theory which is supposed to be valid from observable inflation onwards cannot apply back to an indefinitely early era. The point at which it breaks down is a matter of intense debate at present. With Einstein gravity, 4-d field theory cannot be valid if the energy density exceeds the Planck scale $M_{\text{P}} \equiv (8\pi G)^{-1/2} = 2.4 \times 10^{18} \text{ GeV}$. This is because quantum physics and general relativity come into conflict at that scale, making it the era when classical spacetime first emerges. More generally, it is supposed that any field theory will be just an effective one, valid when relevant energy scales are below some ‘ultra-violet cutoff’ Λ . Above the cutoff, the field theory will be replaced either by

² As usual $a(t)$ is the scale factor of the Universe and $H \equiv \dot{a}/a$ is the Hubble parameter.

a more complete field theory, or by a completely different theory which is generally assumed to be string theory.

The measured values of the gauge couplings suggest the existence of a GUT theory, implying that field theory holds at least up to 10^{16} GeV. This has not prevented the community from considering the possibility that field theory fails at a much lower energy. The idea is that 4-d spacetime would emerge as an approximation to the 10-d spacetime within which string theory is supposed to hold. String theory is formulated in terms of fundamental strings (F strings), but nowadays an important role is supposed to be played by what are called D- p branes (or just D branes) with various space dimensions $0 < p \leq 9$. The electromagnetic, weak and strong forces that we experience might be confined to a particular D-3 brane, while gravity is able to penetrate to the region outside known as the bulk. An important role may be played by D strings, which are D branes with just one of our space dimensions.

3 The initial condition for observable inflation

The models of inflation that we are going to consider apply to at least the last 50 e -folds or so, starting with the exit from the horizon of the observable Universe. One may call this the era of observable inflation, because it is directly constrained by observation through the perturbations which it generates. Assuming Einstein gravity, observable inflation has to take place with energy density $\rho \lesssim (10^{-2} M_P)^4$ or primordial gravitational would have been detected.

The era before observable inflation is not directly accessible to observation, but one may still ask about that era. In particular one may ask how the inflaton field arrives at the starting point for observable inflation. Though not compulsory, it normally is imagined that inflation begins promptly with the emergence of 4-d spacetime. This is indeed desirable for two reasons. One is to prevent the observable Universe from collapsing if the density parameter Ω is initially bigger than 1 (without being fine-tuned to a value extremely close to 1). The other, which applies also to the case $\Omega < 1$, is that inflation protects an initially homogeneous region from invasion by its inhomogeneous surroundings. This is because the event horizon which represents the farthest distance that an inhomogeneity can travel, is finite during inflation. If the onset of inflation is significantly delayed, one would need either a huge initially homogeneous patch or [18] a periodic universe. In contrast, if inflation begins promptly with the emergence of 4-d spacetime, the initially homogeneous region is safe provided only that it is bigger than the event horizon. For almost-exponential inflation the event horizon is of order the Hubble distance.

A simple hypothesis about the emergence of 4-d spacetime was made in [9]. Working in the context of Einstein gravity, the energy density of the Universe at the Planck scale is supposed to be dominated by scalar fields, with the potential in some regions of order M_P^4 and flat enough for inflation to occur there. This setup was termed chaotic inflation, and as an example the potentials $V(\phi) \propto \phi^2$ and ϕ^4 were considered. These are generally called chaotic inflation potentials, but the proposal of [9] regarding the initial condition does not rely on a specific form for the potential. It *is* necessary though that there are regions of field space where the potential is at the Planck scale and capable of inflating. No example of such a potential has been derived from string theory.

An alternative to the chaotic inflation proposal is that inflation begins at the top of a hill in the potential, whose height is much less than M_{P}^4 . In particular, the height could be $\lesssim (10^{16} \text{ GeV})^4$, allowing observable inflation to take place near the hilltop. This proposal is viable even if the process by which the field arrives at the hilltop is very improbable (such as the process of quantum tunneling through a potential barrier), because inflation starting sufficiently near the hilltop gives what is called eternal inflation [19, 20].

During eternal inflation, the volume of the inflating region grows indefinitely, and it can plausibly be argued that this indefinitely large volume outweighs any finite initial improbability. Taking into account the quantum fluctuation, it can be shown [21] that eternal inflation takes place near a hilltop provided that $|\eta| < 6$ where $\eta \equiv V''/3H^2$.

Eternal inflation near a hilltop has been called topological eternal inflation [22]. More generally, eternal inflation occurs whenever the potential over a sufficient range satisfies

$$\left(\frac{H^2}{2\pi\dot{\phi}_{\text{class}}} \right)^2 = \frac{1}{12\pi^2} \frac{V^3}{M_{\text{P}}^6 V'^2} > 1. \quad (1)$$

Here $\dot{\phi}_{\text{class}} = -V'/3H$ is the slow-roll approximation, excluding the stochastic [23] quantum fluctuation $H/2\pi$ per Hubble time. When the left-hand side of Eq. (1) is bigger than 1 the fluctuation dominates so that it can overcome the slow-roll behaviour for an indefinitely long time, during which eternal inflation occurs. In the opposite regime, the fluctuation is small and the left-hand side of Eq. (1) becomes the spectrum of the curvature perturbation. Eternal inflation occurs with the chaotic inflation potential $V \propto \phi^p$, for sufficiently-large field values [24].

Eternal inflation provides a realization of the multiverse idea, according to which all possible universes consistent with fundamental theory (nowadays, string theory) will actually exist [20, 24]. This is because eternal inflation can be of indefinitely long duration, allowing time for tunneling to all local minima of the scalar field potential.

4 Slow-roll inflation

4.1 Basic equations

We will find it useful to classify the models according to the variation $\Delta\phi$ of the inflaton field after the observable Universe leaves the horizon. We will call a model small-field if $\Delta\phi \ll M_{\text{P}}$, medium-field if $\Delta\phi \sim M_{\text{P}}$ and large-field if $\Delta\phi \gg M_{\text{P}}$. Hybrid inflation models are usually constructed to be of the small-field type, the idea being to make close contact with particle physics which is hardly possible for medium- and large-field models.

The inflaton field equation is

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \quad (2)$$

Except near a maximum of the potential (or minimum in the case of hybrid inflation) a significant amount of inflation can hardly occur unless this equation is well-approximated by

$$3H\dot{\phi} \cong -V', \quad (3)$$

with the energy density $3M_{\text{P}}^2 H^2 = V + \frac{1}{2}\dot{\phi}^2$ slowly varying on the Hubble timescale:

$$\dot{H} \ll H^2. \quad (4)$$

Eqs. (3) and (4) together define the slow-roll approximation, and we will use \cong to denote equalities which become exact in that approximation.

Consistency of Eq. (3) with the exact equation requires

$$3M_{\text{P}}^2 H^2 \cong V. \quad (5)$$

and the flatness conditions

$$\epsilon \ll 1 \quad |\eta| \ll 1, \quad (6)$$

where

$$\epsilon \equiv \frac{1}{2}M_{\text{P}}^2(V'/V)^2 \quad \eta \equiv M_{\text{P}}^2 V''/V \quad (7)$$

Requiring that successively higher derivatives of the two sides of Eq. (3) are equal to good accuracy gives more flatness conditions involving more slow-roll parameters. The first two are

$$|\xi^2| \ll 1, \quad \xi^2 \equiv M_{\text{P}}^4 \frac{V'(d^3 V/d\phi^3)}{V^2}, \quad (8)$$

$$|\sigma^3| \ll 1, \quad \sigma^3 \equiv M_{\text{P}}^6 \frac{V'^2(d^4 V/d\phi^4)}{V^3}. \quad (9)$$

The general expression is

$$|\beta_{(n)}^n| \ll 1, \quad \beta_{(n)}^n \equiv M_{\text{P}}^{2n} \frac{V'^{n-1}(d^{n+1} V/d\phi^{n+1})}{V^n}, \quad (10)$$

but only ξ^2 and σ^3 are ever invoked in practice.

It is obvious that these additional parameters can have either sign. The motivation for writing them as powers comes from some simple forms for V , which make $|\xi|$, $|\sigma|$ and $|\beta_{(n)}|$ at most of order η . For more general potentials one can check case-by-case how small are ξ^2 and σ^3 . Usually there is at least a hierarchy

$$\eta \gg \xi^2 \gg \sigma^3 \dots, \quad (11)$$

but slow-roll *per se* requires only that all of the slow-roll parameters are $\ll 1$ and does not require any hierarchy.

A convenient time variable is $N(t)$, the number of e -folds of expansion occurring after some initial time, given by $dN = -Hdt$. In the slow-roll approximation

$$H' \cong -\epsilon H \quad (12)$$

$$\epsilon' \cong 2\epsilon(2\epsilon - \eta) \quad (13)$$

$$\eta' \cong 2\epsilon\eta - \xi^2, \quad (14)$$

$$\xi' \cong 4\epsilon\xi^2 - \eta\xi^2 - \sigma^3, \quad (15)$$

and so on, where a prime denotes d/dN . The first relation says that almost-exponential occurs. The second relation says that ϵ varies slowly. Slow-roll does not guarantee that the other parameters are slowly varying, though this is guaranteed in the usual case that the hierarchy (11) holds.

The flatness conditions are obtained by successive differentiations of the slow-roll approximation. Strictly speaking, a differentiation might incur large errors so that η or higher slow-roll parameters fail to be small (compared with 1). In practice though one expects at least the first few slow-roll parameters to be small.

4.2 Number of e -folds

To obtain the predictions, one needs the scale $k(\phi)$ leaving the horizon when ϕ has a given value. The number of e -folds from then until the end of slow-roll inflation at ϕ_{end} is

$$N(k) \cong M_{\text{P}}^{-2} \int_{\phi_{\text{end}}}^{\phi} \left(\frac{V}{V'} \right) d\phi = M_{\text{P}}^{-1} \left| \int_{\phi_{\text{end}}}^{\phi} \frac{d\phi}{\sqrt{2\epsilon(\phi)}} \right|. \quad (16)$$

For definiteness we will evaluate the predictions for the biggest cosmological scale $k = a_0 H_0$, where the subscript 0 denoted the present epoch, and denote $N(a_0 H_0)$ simply by N . The prediction for any other scale can be obtained using

$$N(k) = N - \ln(k/H_0) \equiv N - \Delta N(k). \quad (17)$$

Taking the shortest cosmological scale to be the one enclosing mass $M = 10^4 M_{\odot}$, those scales span a range $\Delta N = 14$.

The value of N depends on the evolution of the scale factor after inflation. With the maximum inflation scale $V^{1/4} = 10^{16}$ GeV and radiation domination from inflation onwards, $N = 61$. Delaying reheating until $T \sim \text{MeV}$, with matter domination before that, reduces this by 14. With the maximum inflation scale it is therefore reasonable to adopt as an estimate

$$N = 54 \pm 7, \quad (18)$$

Reducing the inflation scale reduces N by $\ln(V^{1/4}/10^{16} \text{ GeV})$, and the lowest scale usually considered is 10^{10} GeV or so, reducing the above central value to 40.

Based on this discussion it seems fair to say that the fractional uncertainty in N is likely to be at most of order 20%. As we shall see, the corresponding uncertainties in the predictions are of the same order in a wide range of models. On the other hand, a very low inflation scale and/or Thermal Inflation [25] could reduce N by an indefinite amount. The only absolute constraint is $N > 14$, required so that perturbations are generated on all cosmological scales. Also, a long era of domination by the kinetic term of a scalar field (kination), corresponding to $P = \rho$, could increase the estimate [26] by up to 14. Taking all of that on board the maximum range would be $14 < N < 75$.

In non-hybrid models, ϵ usually increases with time and inflation ends when one of the flatness conditions fails, after which ϕ goes to its vacuum expectation value (vev). From its definition, ϵ increasing with time corresponds to $\ln V$ being concave-downward. In this case, the value of ϕ_* obtained from Eq. (16) will typically be insensitive to ϕ_{end} , making the model more predictive.

In some hybrid models, ϵ decreases with time ($\ln V$ concave-upward), and inflation ends only when the waterfall field is destabilized. In other hybrid inflation models though, ϵ increases with time ($\ln V$ concave-downward), and *slow-roll* inflation may end before the waterfall field is destabilized through the failure of one of the flatness conditions. If that happens, a few more e -folds of inflation can take place

while the inflaton oscillates about its vev (locked inflation [27]), until the amplitude of the oscillation becomes low enough to destabilize the waterfall field.

4.3 Predictions

The vacuum fluctuation of the inflaton generates a practically gaussian perturbation, with spectrum $\mathcal{P}_\phi(k) = (H_k/2\pi)^2$ where the subscript k indicates horizon exit $k = aH$. This perturbation generates a time-independent curvature perturbation with spectrum [7]

$$\mathcal{P}_\zeta(k) = \frac{1}{24\pi^2 M_{\text{P}}^4} \frac{V_k}{\epsilon_k}. \quad (19)$$

The error in this estimate will come from the error in \mathcal{P}_ϕ and the error in the slow-roll approximation. Both are expected to give a small fractional error, of order $\max\{\epsilon, \eta\}$. Differentiating with respect to $\ln k$ to get the spectral index may incur a fractional error $\gtrsim 1$ if η is rapidly varying [28], but that is not the case in the usual models. Differentiating Eq. (19) using Eqs. (12) and (13) give the spectral tilt;

$$n - 1 \equiv d \ln \mathcal{P}_\zeta / d \ln k = 2\eta_k - 6\epsilon_k. \quad (20)$$

If in addition $d\eta/dN$ (equivalently, ξ^2) is slowly varying this may be differentiated again to obtain the running,

$$dn/d \ln k = -16\epsilon\eta + 24\epsilon^2 + 2\xi^2. \quad (21)$$

Observable inflation can take place near a maximum or minimum of the potential even with the flatness condition $|\eta| \ll 1$ mildly violated to become $|\eta| \sim 1$ (so-called fast-roll inflation [29], though note that $\dot{\phi}$ is still small making H almost constant).³ This quite natural possibility would give tilt $|n - 1| \simeq 1$, which is also quite compatible with the original arguments of Harrison [30] and Zeldovich [31] for $n \sim 1$ and all known environmental arguments. The very small tilt now observed is not required by any general consideration, and a large tilt $n - 1 \sim -0.3$ had previously been considered as a serious possibility to make a critical-density CDM model more viable [11].

During inflation, the vacuum fluctuation generates a primordial tensor perturbation, setting the initial amplitude for gravitational waves which oscillate after horizon entry. The spectrum \mathcal{P}_T of this perturbation is conveniently specified by the tensor fraction $r \equiv \mathcal{P}_T/\mathcal{P}_\zeta$. In the slow-roll approximation [11],⁴

$$r = 16\epsilon = -8n_T, \quad (22)$$

where $n_T \equiv d \ln \mathcal{P}_T / d \ln k$. The second relation has become known as the consistency condition, and its violation would show that the curvature perturbation is not generated by a single-field slow-roll inflation.

Using the observed value for the spectrum of the curvature perturbation, the tensor fraction is given by

³ Very close to a maximum is the regime of eternal inflation, which presumably precedes fast- or slow-roll inflation.

⁴ The definition of r in this reference was slightly different.

$$r = \left(\frac{V^{1/4}}{3.3 \times 10^{16} \text{ GeV}} \right)^4. \quad (23)$$

The tensor fraction can also be related to $\Delta\phi$. Suppose that slow-roll persists to almost the end of inflation and that $\ln V$ is concave-downward throughout. Then $|V/V'|$ is continuously increasing, and Eq. (16) gives $2\epsilon < N^{-2}(\Delta\phi/M_P)^2$. This can be written [32, 3]

$$16\epsilon = r < 0.003 \left(\frac{50}{N} \right)^2 \left(\frac{\Delta\phi}{M_P} \right)^2. \quad (24)$$

Now suppose instead that slow-roll persists to the end of inflation, without any requirement on the shape of the potential. As a consequence of slow-roll, ϵ varies little during one Hubble time and there are only 50 or so Hubble times. It follows that one may expect ϵ to be at least roughly constant, in which case the right hand side of Eq. (24) provides at least a rough estimate of the actual value of $\Delta\phi$.

Finally, let us adopt the most conservative possible position and consider just the change $\Delta\phi_4$ during the four e -folds after the observable Universe leaves the horizon, that being the era when an observable tensor perturbation may actually be generated. Then it is certainly safe to assume that ϵ has negligible variation, leading to the quite firm estimate

$$r \simeq \frac{1}{2} \left(\frac{\Delta\phi_4}{M_P} \right)^2. \quad (25)$$

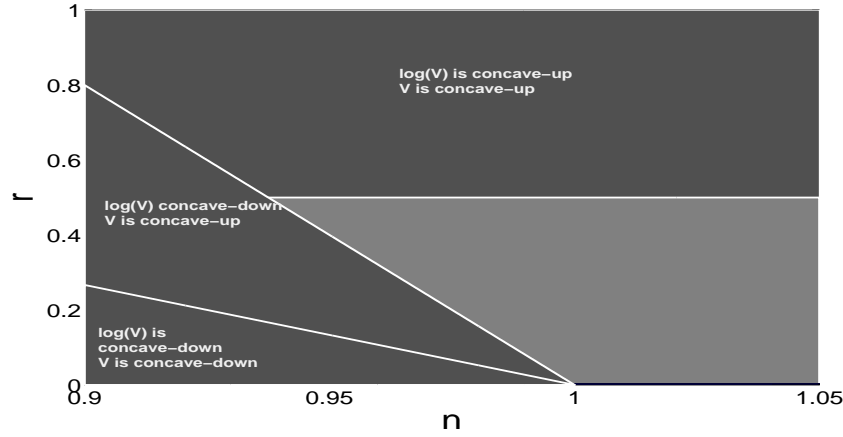


Fig. 1.

In Figure 1, the r - n plane is divided into three regions, according to whether V and $\ln V$ are concave-upward or concave-downward while cosmological scales leave the horizon. Figure 2 repeats the plot in the $\ln r$ - n plane.

If the concave-upward -downward behaviour persists till the end of slow-roll inflation, the right-hand region is inhabited exclusively by hybrid inflation models, since otherwise inflation would never end. With that assumption, Eqs. (24) and (25)

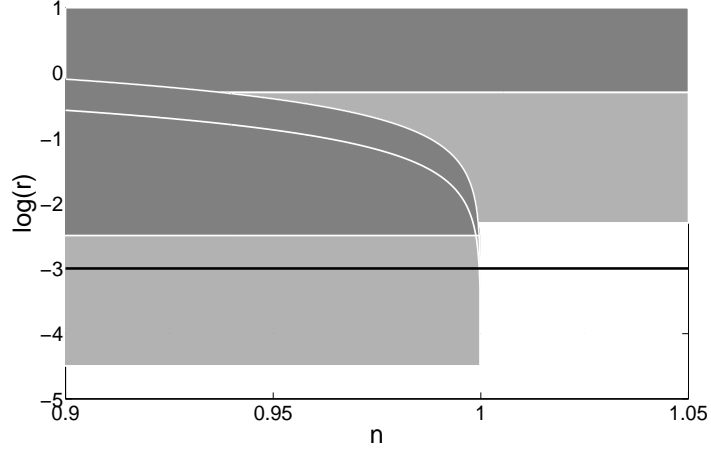


Fig. 2.

imply that the lightly-shaded region of the Figures is excluded if $\Delta\phi > 0.1M_{\text{P}}$, and that the heavily-shaded region is excluded if $\Delta\phi > M_{\text{P}}$. (In the right-hand region, corresponding to concave-upward $\ln V$, we used Eq. (25) with $\Delta\phi_4 = \Delta\phi$; the actual bound will be tighter since in reality $\Delta\phi_4 < \Delta\phi$.)

4.4 Observational constraints

According to observation [33] value of the spectrum \mathcal{P}_ζ has the almost scale-invariant value $(5 \times 10^{-5})^2$, with negligible error. This gives the constraint

$$V^{1/4}/\epsilon^{1/4} = 0.027M_{\text{P}} = 6.6 \times 10^{16} \text{ GeV}, \quad (26)$$

which we will call the cmb constraint.

Setting $r = 0$ and taking n to be scale-independent, observation gives [33] $n \simeq 0.948^{+0.015}_{-0.018}$. Allowing r and a scale-independent $dn/d \ln k$ gives a higher n and $n' \simeq -0.10 \pm 0.05$, consistent with no running at 2σ level. The allowed region in the r - n plane is shown in Figure 3. (This is a corrected version of the Figure in [33], kindly supplied by the authors). The bound $r = 0$ is seen to apply for $r \ll 0.1$.⁵ Within a few years there will be either a detection of r or a bound $r < 10^{-2}$. If r is below 10^{-3} it will probably be undetectable by any means. This value is marked in Figure 2.

From all this, we see that small- and medium-field generally give $r \lesssim 10^{-2}$. This means that the predicted tensor fraction is unlikely to be observed. It also means that the prediction for the spectral tilt can be taken as simply $n - 1 = 2\eta$; to reproduce the observed negative tilt the potential of a small- or medium-field model should be concave-downwards while cosmological scales leave the horizon.

⁵ The 1- σ limit with r set equal to zero is tighter than the limit read off from setting $r = 0$ in the r - n plot, because the joint probability distribution is non-gaussian

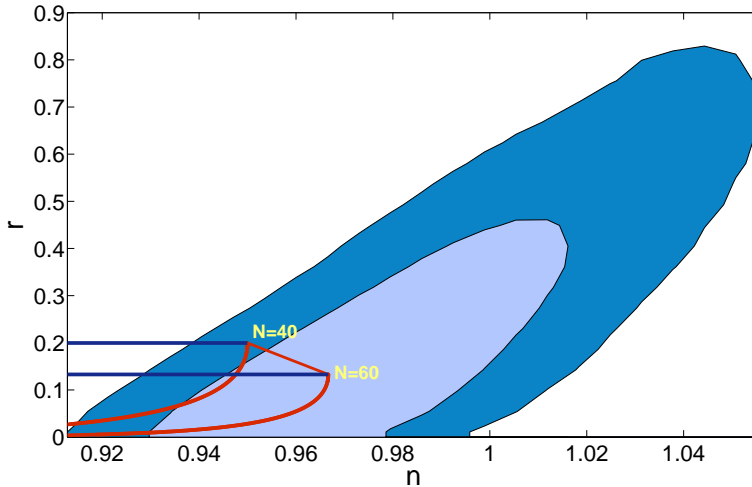


Fig. 3. The closed areas show the regions allowed by observation at 66% and 95% confidence levels. The curved lines are the Natural Inflation predictions for $N = 20$ and $N = 75$, and the horizontal lines are the corresponding multi-field Chaotic Inflation predictions. The junction of each pair of lines corresponds to single-field Chaotic Inflation.

4.5 Beyond the standard paradigm

Throughout we have adopted the standard paradigm, whereby the curvature perturbation ζ is generated by the inflaton perturbation in a single-field slow-roll inflation model. In general there will exist other light fields, each possessing a perturbation with the nearly flat spectrum $(H/2\pi)^2$, any one of which might be responsible for the curvature perturbation.

The predictions in this more general scenario are best calculated through the δN formalism [35, 36, 37, 38, 39]. As our main focus is on the standard paradigm we just give some basic results without derivation. It is convenient to use at horizon exit a field basis $\{\phi, \sigma_i\}$, where ϕ points along the inflaton trajectory and the σ_i ($i = 2 \cdots M$) are orthogonal. The perturbation $\delta\phi$ then generates the same time-independent curvature perturbation as in the single-field case, whose spectrum we denote by \mathcal{P}_{ζ_ϕ} . The orthogonal perturbations give no contribution to the curvature at horizon exit, but one or more of them may generate an additional contribution later which may be dominant by the time that the curvature perturbation settles down to the final time-independent value (obtaining as cosmological scales start to approach the horizon) whose spectrum we denote simply by \mathcal{P}_ζ . The additional contribution may be generated during inflation in which case we are dealing with a multi-field inflation model, or later through for example the curvaton mechanism [40]. In the latter case, the model of inflation is irrelevant; all that matters is that the Hubble parameter is slowly varying. Liberated from the constraint to generate the curvature perturbation, model-building becomes much easier [86].

The cmb normalization (26) now becomes an upper bound, implying a lower inflation scale for a given value of ϵ . The spectral index in general depends on the evolution after horizon exit [36, 1], but in the most natural case that the contribution of single orthogonal field $\sigma \equiv \sigma_i$ dominates it is given by the potential at horizon exit as

$$n(k) - 1 = 2\eta_{\sigma\sigma} - 2\epsilon, \quad (27)$$

where $\eta_{\sigma\sigma} = \partial^2 V / \partial \sigma^2$. (The case that two contributions are comparable may arise by accident, or in special models where ϕ and an orthogonal field are related such as the one involving axion physics which is described in [41].)

Since the tensor perturbation depends only on H the tensor fraction r is reduced;

$$r = 16\epsilon \frac{\mathcal{P}_{\zeta_\phi}}{\mathcal{P}_\zeta} < 16\epsilon. \quad (28)$$

It is negligible if an orthogonal contribution dominates.

We did not mention non-gaussianity. According to the standard paradigm, the non-gaussianity is [42] about 100 times smaller than the level that can be detected from the cmb anisotropy (and/or galaxy surveys) though it has recently been claimed [43] that a measurement from the 21-cm anisotropy might be possible. In contrast, non-standard paradigms may easily generate non-gaussianity at an observable level; in particular the curvaton and inhomogeneous reheating scenarios are expected to generate non-gaussianity at a level that is at least marginally observable through the cmb. If non-gaussianity is observed we will be dealing with functions (of rotationally-invariant scalars formed from the wave-vectors that define the bispectrum, trispectrum etc.) as opposed to numbers, which will provide powerful information about the origin of the curvature perturbation.

All of this assumes slow-roll inflation. That possibility is compatible with the simultaneous detection of a tensor perturbation *and* non-gaussianity only if some orthogonal field can generate the non-gaussianity without being dominant (a highly constrained scenario [44]). The main alternative to slow-roll inflaton seems to be inflation with non-quadratic kinetic terms, called k -inflation [45], of which special forms are the brane world DBI inflation scenario [46] and ghost inflation [47].

5 Modular inflation

We begin our survey of inflation models with the most plausible medium-field model, which goes by the name of modular inflation. This is a non-hybrid model in which the inflaton is a modulus. It was suggested a long time ago [34] and its possible realization in the context of brane worlds is under investigation at present.

Moduli may play other roles too in the early Universe, and we describe their properties before getting to the inflation model. For the present purpose a modulus may be defined as a field with a potential of the form

$$V = V_0 f(\phi/M_P), \quad (29)$$

This is supposed to hold in the range $0 < \phi \lesssim M_P$, with the function $f(x)$ and its low derivatives of order 1 at a generic point. At the vev, where f and f' vanish, the mass-squared $m^2 \equiv V''$ is typically of order V_0/M_P^2 . If the potential has a maximum,

it will typically be located at a distance of order M_{P} from the vev with the tachyonic mass-squared V'' typically of order $-m^2$.

Fields with this property are expected (though not inevitable) in a field theory derived from string theory. Usually the field theory is taken to be supersymmetric though moduli are expected anyway. Moduli are usually supposed to have interactions of only gravitational strength, corresponding to a lifetime $\Gamma \sim m^3/M_{\text{P}}^2$. Alternatively though, a modulus may have interactions of ordinary strength, in particular gauge interactions. The fixed point of the symmetry group is then called a point of enhanced symmetry. Such a point might correspond to either the vev or to a maximum of the potential. It may even be possible for both of these to be points of enhanced symmetry, involving different symmetry groups.

Moduli may affect cosmology in several ways. Usually they are considered in the context of supersymmetry, and the simplest expectation for the mass is then $m \sim \text{TeV}$, corresponding to what we may call light moduli. A light modulus is typically displaced strongly from its vev during inflation, by an amount which puts its subsequent oscillation and gravitational-strength decay into conflict with nucleosynthesis. To avoid this ‘moduli problem’ one may suppose that all moduli are heavy, or that there is Thermal Inflation [25].

Now we turn to modular inflation. It is usually supposed to take place near a maximum or saddle-point of the potential, with just one modulus ϕ varying significantly. As many moduli typically exist, that may not be easy to arrange. Supposing that it happens let us set $\phi = 0$ at the maximum and consider the power series for the potential. The generic expectation would be for the quadratic term alone to provide at least a crude approximation to the potential in the slow-roll regime, corresponding to

$$V(\phi) = V_0 \left(1 + \frac{1}{2} \eta_0 \frac{\phi^2}{M_{\text{P}}^2} \right) \quad (30)$$

But this requires (from Eq. (29)) roughly $\eta_0 \sim -1$ which gives spectral tilt $n - 1 \sim -1$ in contradiction with observation. To provide a modular inflation model one suppresses the quadratic term, either by means of a symmetry [48] or more usually by fine tuning (see for instance [49]).

If the suppressed quadratic term is still required to dominate while cosmological scales leave the horizon, one obtains the scale-independent prediction $n = 1 + 2\eta_0$ which can agree with observation by choice of η_0 . This prediction is scale-independent which might in the future allow it to be distinguished from other predictions for n . Of course, one has to invoke additional terms to end inflation, presumably at a value $\phi_{\text{end}} \sim M_{\text{P}}$. The tensor fraction is

$$r = 2 \left(\frac{\phi_{\text{end}}}{M_{\text{P}}} \right)^2 (1 - n)^2 e^{-N(1-n)} \sim 10^{-3.5} \left(\frac{\phi_{\text{end}}}{M_{\text{P}}} \right)^2. \quad (31)$$

Taking $\phi_{\text{end}} \sim M_{\text{P}}$ gives the result shown in Figure 10. The tensor fraction is unobservable, but corresponds to a high normalization scale $V^{1/4} \sim 10^{15} \text{ GeV}$, meaning that we are not dealing with a light modulus.

It is more reasonable to suppose that the suppressed quadratic term is negligible. Then, as a rough approximation it may be reasonable to write

$$V \simeq V_0 \left[1 - \left(\frac{\phi}{\mu} \right)^p \right], \quad (32)$$

with $p \gtrsim 3$ (not necessarily and integer) and $\mu \sim M_{\text{P}}$.

If this approximation holds for some reasonable length of time after cosmological scales leave the horizon it gives

$$\phi_*^{p-2} = [p(p-2)\mu^{-p}NM_{\text{P}}^2]^{-1}, \quad (33)$$

(independently of ϕ_{end}) and

$$n-1 = -\frac{2}{N} \left(\frac{p-1}{p-2} \right). \quad (34)$$

For the range $3 < p < \infty$ with $N = 50$ we get $0.92 < n < 0.96$. The cmb normalization corresponds to a tensor fraction

$$r \simeq \frac{0.001}{(p-2)^4} \left(\frac{\mu}{M_{\text{P}}} \right)^{\frac{p}{2p-4}} \left(\frac{50}{N} \right)^{\frac{2(p-1)}{p-2}}. \quad (35)$$

This is shown in Figure 10 with $\mu = M_{\text{P}}$. Again, the tensor fraction is too small to detect but still corresponds to a high energy scale $V^{1/4} \sim 10^{15}$ GeV. These estimates agree to rough order of magnitude with results obtained numerically using potentials derived from string theory (see for instance [49]).

6 Small-field models

A range of small-field models has been proposed. Before describing them we make some general remarks, followed by a very basic treatment of supersymmetry which is invoked in most small-field models.

The motivation for small-field models comes from ideas about what is likely to lie beyond the Standard Model of particle physics. Choosing the origin as the fixed point of the relevant symmetries, the tree-level potential will have a power series expansion,

$$V(\phi) = V_0 \pm \frac{1}{2}m^2\phi^2 + M\phi^3 + \frac{1}{4}\lambda\phi^4 + \sum_{d=5}^{\infty} \lambda_d M_{\text{P}}^4 \left(\frac{\phi}{M_{\text{P}}} \right)^d \quad (36)$$

The lower-order terms of Eq. (36), which do not involve M_{P} , are renormalizable terms (corresponding to a renormalizable quantum field theory). The higher-order terms, which disappear in the limit $M_{\text{P}} \rightarrow \infty$, are non-renormalizable terms. We are taking m^2 positive and as indicated the quadratic term might have either sign. The other renormalizable terms will usually be positive, but the non-renormalizable terms might have either sign.

According to a widely-held view, non-renormalizable terms of arbitrarily high order are expected, with magnitudes big enough to place this expansion out of control at $\phi \gtrsim M_{\text{P}}$. The typical expectation is $|\lambda_d| \sim 1$ if M_{P} is the ultra-violet cutoff and $|\lambda_d| \sim (M_{\text{P}}/\Lambda)^d$ (the latter corresponding to the replacement $M_{\text{P}} \rightarrow \Lambda$) if the cutoff Λ is smaller. This view is part of a more general one, according to which the lagrangian of a field theory ought to contain all terms that are allowed by the symmetries, with coefficients typically of order 1 in units of the ultra-violet cutoff (see for instance [50]).

If the field theory is replaced by a more complete field theory above the cutoff, the λ_d can be calculated and will be of the advertised order of magnitude if ϕ has unsuppressed interactions. But if instead it is replaced by string theory above the cutoff, then estimates of λ_d should come from string theory. Such estimates are at present not available, except for moduli.⁶ In general then, one is free to accept or not the usual view about non-renormalizable terms.⁷

Following [1], let us see what sort of conditions the terms in Eq. (36) must satisfy, to achieve inflation in the small-field regime $\phi \ll M_P$. We discount the possibility of extremely accurate cancellations between different terms. This means that the constant term has to dominate, and that we require the addition of any one other term to respect the flatness condition $|\eta| \ll 1$, the other flatness conditions then being automatic.⁸

We shall not consider the cubic term, which usually is forbidden by a symmetry. For the other terms $|\eta| \ll 1$ is equivalent to

$$m^2 \ll \frac{V_0}{M_P^2} \simeq 3H_*^2 \quad (37)$$

$$\lambda \ll \frac{V_0}{M_P^4} \frac{M_P^2}{\phi^2} \quad (38)$$

$$\lambda_d \ll \frac{V_0}{M_P^4} \left(\frac{M_P^2}{\phi^2} \right)^{\frac{d-2}{2}}. \quad (39)$$

One might think that the second and third conditions can always be satisfied by making ϕ small enough, but this is not correct because there is a lower limit on the variation of ϕ . Indeed, during just the ten or so e -folds while cosmological scales leave the horizon Eqs. (16) and (26) require ϕ to change by at least $10^4 V^{1/2}/M_P$ and ϕ cannot be smaller than that on all such scales. We conclude that

$$\lambda \lesssim 10^{-8} \quad (40)$$

$$\lambda_d \lesssim 10^{-8} \left(\frac{10^{16} \text{ GeV}}{V_0^{1/4}} \right)^{2(d-4)}. \quad (41)$$

⁶ In the case of moduli Eq. (29) implies a strong suppression of the couplings. However, the inflaton in a small-field model is not usually supposed to be a modulus because the origin in small-field models is usually taken to be the fixed point of the symmetries of some unsuppressed interactions, which would make the origin a point of enhanced symmetry for the modulus.

⁷ This is less true if supergravity is invoked because the non-renormalizable terms are then present and out of control for generic choices of the functions defining the theory. But one can still make special choices to avoid the problem.

⁸ In a supersymmetric theory one instead consider A -term inflation [51, 52]. Dropping the constant term V_0 , one can choose a flat direction (say in the space of the MSSM scalars) in which the leading non-renormalizable term in the superpotential generates an A -term. Then a fine-tuned match between three terms in the potential can give $V' = V'' = 0$ for a particular field value. Inflation can then take place near that value and naturally reproduce the cmb normalization. By a suitable choice of the fine-tuning it can also reproduce the observed spectral index, though it can also give any value in the slow-roll range $0 \lesssim n \lesssim 2$ [52].

The first condition requires λ to be very small, and the second condition requires at least the first few λ_d to be very small unless the inflation scale is well below 10^{16} GeV. Supersymmetry can ensure these conditions, either by itself or combined with an internal symmetry. Alternatively one can invoke just an internal symmetry corresponding to $\phi \rightarrow \phi + \text{const}$, making ϕ a PNCB, though as we remark later that is not so easy to arrange as one might think.

Finally, we recall that for a generic field in an effective field theory, M_P in Eq. (36) might be replaced by an ultra-violet cutoff $\Lambda_{UV} < M_P$, arising either because heavy fields have been integrated out, or because large extra dimensions come into play. One hopes that such a thing does not happen for the inflaton field, because it would make it more difficult to satisfy the flatness conditions [53]. Fortunately, the presence of large extra dimensions does not in itself prevent M_P from being the effective cutoff for at least some of the fields.

7 Supersymmetry: general features

Field theory beyond the Standard Model is usually required to possess supersymmetry. Supersymmetry [54] is an extension of Lorentz invariance. Its outstanding prediction is that each fermion should have bosonic superpartners, and vice versa, with identical mass and couplings in the limit of unbroken supersymmetry. Supersymmetry has to be broken in our Universe.

Supersymmetry is usually taken to be a local symmetry, and is then called supergravity because it automatically incorporates gravity.⁹ In that case the breaking is spontaneous. In many situations, global supersymmetry is used with the expectation that it will provide a good approximation to supergravity. In that case the breaking can be spontaneous and/or explicit.

We shall deal with the simplest version of supersymmetry, known as $N = 1$ supersymmetry, which alone seems able to provide a viable extension of the Standard Model. Here, each spin-half field is paired with either a complex spin-zero field (making a chiral supermultiplet), or else with a gauge boson field (making a gauge supermultiplet). With supergravity, the graviton (spin two) comes with a gravitino (spin 3/2). With spontaneously broken global supersymmetry there is instead a spin 1/2 goldstino.

One motivation for supersymmetry concerns the mass of the Higgs particle, given by the vev of $\partial^2 V / \partial \phi^2$ where ϕ is the Higgs field. The function V that we have up till now been calling simply the potential is only an effective one, and not the ‘bare’ potential entering into the lagrangian which defines the field theory. Interactions of the scalar fields with themselves and each other change the bare potential into an effective potential. We will be concerned with perturbative quantum effects represented by Feynman diagrams. If we including just tree-level (no-loop) diagrams, the effective potential is still given by the power series (36) with different (renormalized) values for the coefficients in the series. Loop corrections give further renormalization of the coefficients, which is our immediate concern. (They also give the potential logarithmic terms that have to be added to the power series, which we come to later.)

⁹ Some brane world scenarios explicitly break local supersymmetry which means there is actually explicitly broken global supersymmetry.

The point now is that the loop ‘correction’ in a generic field theory will be large, driving the physical mass up to a value of order the ultra-violet cutoff. As the latter is usually supposed to be many orders of magnitude above the physical Higgs mass, one must in the absence of supersymmetry fine-tune the bare mass so that it almost exactly cancels the loop correction. To protect the Higgs mass from this fine tuning, one needs to keep the loop correction under control by means of a symmetry which would make it zero in the unbroken limit. The best symmetry for doing that job in the case of the Higgs field is supersymmetry.¹⁰

In a supersymmetric extension of the Standard Model, each particle species must come with a superpartner. It turns out that at least two Higgs fields are then needed. Keeping just two, one arrives at the Minimal Supersymmetric Standard Model (MSSM), which is a globally supersymmetric theory with canonically-normalized fields. The partners of the quarks and leptons are called squarks and sleptons, those of the Higgs fields are called higgsinos, and those of the gauge fields are called gauginos.

Unbroken supersymmetry would require that each Standard Model particle has the same mass as its partner. This is not observed, which means that the global supersymmetry possessed by the MSSM must be broken in the present vacuum. To agree with observation it turns out that the breaking has to be explicit as opposed to spontaneous. To ensure that supersymmetry continues to do its job of stabilizing the potential against loop corrections, the breaking must be of a special kind called soft breaking. Soft supersymmetry breaking has to give slepton and squark masses very roughly of order 100 GeV. They cannot be much smaller or they would have been observed, and they cannot be much more bigger if supersymmetry is to do its job of stabilizing the Higgs mass.

Softly broken supersymmetry explains with high accuracy the observed ratio of the three gauge couplings (determining the strengths of the strong, weak and electromagnetic interactions) on the hypothesis that there is a GUT. This feature is actually preserved if one allows the squarks and sleptons to be extremely heavy (hence not observable), a proposal known as Split Supersymmetry.

The LHC will soon determine the nature of the fundamental interactions immediately beyond the Standard Model, and may or may not find evidence for supersymmetry. In the latter case we will know that supersymmetry is too badly broken to be relevant for the Standard Model. It might still be relevant in the early Universe and in particular during inflation, but there is no doubt that increased emphasis will then be placed on non-supersymmetric inflation models. A good candidate for non-supersymmetric inflation would be modular inflation. Alternatively, one might make the inflaton a PNCB, or just accept extreme fine tuning.

¹⁰ If a symmetry other than supersymmetry were to be used, the Higgs field ϕ would become a PNCB corresponding to a shift symmetry $\phi \rightarrow \phi + \text{constant}$. It is difficult for a shift symmetry to protect the Higgs mass, because the symmetry will be broken by the strong couplings that the Higgs is known to possess. This problem can be overcome by what is called the Little Higgs mechanism but the resulting schemes are complicated especially if the ultra-violet cutoff is supposed to be many orders of magnitude bigger than the observed mass.

8 Supersymmetry: form of the potential

In a supergravity theory, the potential is a function of the complex scalar fields, of the form

$$V(\phi_i) = V_+(\phi_i) - 3M_{\text{P}}^2 m_{3/2}^2(\phi_i) \quad \text{vsugra07} \quad . \quad (42)$$

The first term is positive, and spontaneously breaks supersymmetry.

In the vacuum, $m_{3/2}(\phi_i)$ becomes the gravitino mass which we denote simply by $m_{3/2}$. Let us denote the vev of the first term by M_{S}^4 . The near-cancellation of the two terms in the vacuum is unexplained (the cosmological constant problem). The explicitly broken global supersymmetry seen in the MSSM sector is supposed to be obtained from the full potential as an approximation. To achieve this the spontaneous breaking must take place in some ‘hidden sector’ with some ‘messenger’ sector communicating (mediating) between the hidden sector and the MSSM sector. The value of M_{S} required to give squark and slepton masses of order 100 GeV depends on the strength of the mediation. Let us characterize it by M_{mess} , with $100 \text{ GeV} = M_{\text{S}}^2/M_{\text{mess}}$. Gravitational-strength mediation (‘gravity mediation’) corresponds to $M_{\text{mess}} \sim M_{\text{P}}$ and the biggest reasonable range is $10^4 \text{ GeV} \lesssim M_{\text{mess}} \lesssim 10^{12}$.¹¹ The corresponding gravitino mass is between 1 eV and 10^6 GeV .

Coming to inflation, supersymmetry stabilizes the potential against loop corrections just as in the MSSM Higgs case. Also, the small λ required in the tree-level potential can be obtained quite naturally. One generally assumes that the first term of Eq. (42) dominates since there is no reason to expect a fine cancellation. Assuming that supersymmetry in the early Universe is broken at least as strongly as in the vacuum, this requires $V \gtrsim M_{\text{S}}^4$. Partly for that reason, very low-scale inflation is difficult to achieve.

Now we come to what has been called the η problem. The supergravity potential can be written as the sum of two terms, called the F term and the D term. In most inflation models V comes from the F term. Then, each scalar field typically has mass-squared at least of order $m^2 \gtrsim V/M_{\text{P}}^2 = 3H^2$. For the inflaton this is in mild conflict with the slow-roll requirement $|\eta| \ll 1$ [55, 58, 56, 57].

Even if we allow the curvature perturbation to be generated after inflation, say in the curvaton model, we still need $m^2 \ll V/M_{\text{P}}^2$ for the curvaton field. In that case there may be a problem even after inflation, because a generic supergravity theory still gives each scalar field an effective mass at least of order H [59] except during radiation domination [60], which will tend to drive each field to its unperturbed value and kill the curvature perturbation.

Returning to the standard scenario for generating the curvature perturbation, we typically need $|\eta| \sim 0.01$ to generate the observed spectral tilt. This represents an order one percent fine-tuning which is not too severe. What is perhaps more serious is that the η problem calls into question the validity of any model which is formulated within the context of global supersymmetry. It is easy to ensure $|\eta| \ll 1$ in such a theory, but having done that the supergravity correction may still be big and completely alter the model. In a typical global supersymmetry model though, the same is true of other types of correction as well.

¹¹ The upper limit corresponds to anomaly mediation, which is gravity mediation suppressed by a loop factor. The lower limit is an interpretation of $M_{\text{S}} \gg 100 \text{ GeV}$, required so that the hidden sector is indeed hidden.

9 One-loop correction

Loop corrections add a logarithmic term to the effective potential. In the direction of any field ϕ , the one-loop correction is

$$\Delta V(\phi) = \sum_i \frac{\pm \mathcal{N}_i}{64\pi^2} M_i^4(\phi) \ln \left[\frac{M_i^2(\phi)}{Q^2} \right]. \quad (43)$$

This is called the Coleman-Weinberg potential. The sum goes over all particle species, with the plus/minus sign for bosons/fermions, and \mathcal{N}_i the number of spin states. The quantity $M_i^2(\phi)$ is the effective mass-squared of the species, in the presence of the constant ϕ field. For a scalar, $M_i^2 = \partial^2 V / \partial \phi_i^2$, which is valid for ϕ itself as well as other scalars.

The quantity Q is called the renormalization scale. If the loop correction were calculated to all orders, the potential would be independent of Q . In a given situation, Q should be set equal to a typical energy scale so as to minimize the size of the loop correction and its accompanying error. Focusing on the inflaton potential, we should set Q equal to a typical value of ϕ (one within the range which corresponds to horizon exit for cosmological scales). That having been done, the *magnitude* of ΔV will typically be negligible, but its derivatives may easily be significant.

If supersymmetry were unbroken, each spin-1/2 field would have a scalar- or gauge field partner with the same mass and couplings, causing the loop correction to vanish. In reality supersymmetry is broken. To see how things work out, let us consider the loop correction from a chiral supermultiplet, consisting of a spin-1/2 particle with a scalar partner. The partner is a complex field $\psi = (\psi_1 + i\psi_2)/\sqrt{2}$, whose real components ψ_i have true masses m_i . If there is an interaction $\frac{1}{2}\lambda'\phi^2|\psi|^2$, this gives $M_i^2 = m_i^2 + \frac{1}{2}\lambda'\phi^2$ ($i = 1, 2$). (We use the prime to distinguish this coupling from the self-coupling λ in the tree-level potential (36) of the inflaton.) The spin-1/2 field typically has true mass $m_f = 0$, and its interaction with ϕ generates an effective mass-squared $M_f^2(\phi) = \frac{1}{2}\lambda'\phi^2$. (This result is not affected by either spontaneous or soft supersymmetry breaking.) When ϕ is much bigger than m_i , the loop correction is therefore

$$\Delta V \simeq \frac{1}{32\pi^2} \left[\sum_{i=1,2} \left(m_i^2 + \frac{1}{2}\lambda'\phi^2 \right)^2 - 2 \left(\frac{1}{2}\lambda'\phi^2 \right)^2 \right] \ln \frac{\phi}{Q}. \quad (44)$$

The coefficient of ϕ^4 vanishes by virtue of the supersymmetry. For the other terms, we will consider two cases. Suppose first that global supersymmetry is spontaneously broken during inflation. Then it turns out that typically $m_1^2 = -m_2^2$, causing the coefficient of ϕ^2 in Eq. (44) to vanish. This leaves

$$\Delta V \simeq \frac{m_1^4}{32\pi^2} \ln \frac{\phi}{Q}. \quad (45)$$

In this case the derivatives of ΔV are independent of Q , making its choice irrelevant as the magnitude of ΔV is negligible.

Now suppose instead that global supersymmetry is explicitly (softly) broken during inflation, the coefficient of ϕ^2 in Eq. (44) does not vanish, but instead typically dominates the constant term. Adding the loop correction to the mass term of the tree-level potential gives

$$\Delta V = \frac{1}{2} \left[m^2 + \frac{\lambda'}{32\pi^2} (m_1^2 + m_2^2) \ln \frac{\phi}{Q} \right] \phi^2, . \quad (46)$$

This expression is valid over a limited range of ϕ , if Q set equal to a value of ϕ within that range. If a large range of ϕ is under consideration, it should be replaced by an expression of the form

$$\Delta V = \frac{1}{2} m^2(\phi) \phi^2. \quad (47)$$

The “running mass” $m^2(\phi)$ is calculated from what are called renormalization group equations (RGEs).

The above discussion involved the loop correction due to a chiral supermultiplet. Couplings involving chiral super multiplets, such as λ' , are called Yukawa couplings and they can be very small. We could instead have discussed the loop correction due to a gauge supermultiplet, consisting of a spin-1/2 field whose partner is a gauge field. The couplings involving gauge super multiplets are called gauge couplings and denoted usually by g . They are not expected to be very small. The loop correction from a gauge supermultiplet is essentially of the above form, with λ' replaced by g .

Finally, if there is no supersymmetry, the loop correction typically destabilizes the tree-level potential, and in particular it gives to the mass of each scalar field a contribution which is typically of order the ultra-violet cutoff. To obtain an acceptable potential, and in particular acceptable masses, one has to invoke a fine-tuned cancellation between the loop correction and the tree-level potential. Considering just the contribution from the spin-1/2 part of Eq. (44), and adding it to the self-coupling of ϕ , one has

$$\Delta V = \frac{1}{4} \left[\lambda - \left(\frac{\lambda'}{4\pi} \right)^2 \ln \frac{\phi}{Q} \right] \phi^4. \quad (48)$$

As with the mass, the RGE's give a more accurate result, corresponding to $\Delta V = \frac{1}{4} \lambda(\phi) \phi^4$ with a running coupling $\lambda(\phi)$.

10 Small-field models: moving away from the origin

In this section we consider small-field potentials with the shape shown in Figure 4. We begin with non-hybrid models, taking the origin as the fixed point of the symmetries. Then the minimum of the potential corresponds to a nonzero vev, and the potential vanishes there. Such models are usually called New Inflation models, since that was the name given to the first viable slow-roll model which happened to be of that kind.

The situation for New Inflation is similar to the one we discussed for modular inflation. Keeping the quadratic term alone cannot be a good approximation throughout inflation. Assuming that the quadratic term is already negligible when cosmological scales leave the horizon, the approximation Eq. (32) seems reasonable, with $p \gtrsim 3$ and now $\mu \ll M_P$. With this approximation the spectral tilt is given by Eq. (34). The tensor fraction is given by Eq. (35) with $\mu \ll M_P$ making it absolutely negligible, and allowing an inflation scale far below 10^{15} GeV.

The original New Inflation model corresponded to $p = 4$;

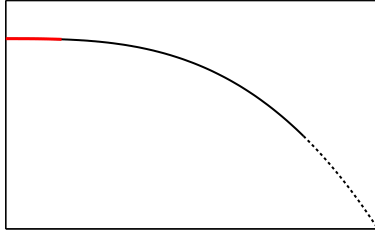


Fig. 4. Modular, new, inverted hybrid, mutated hybrid.

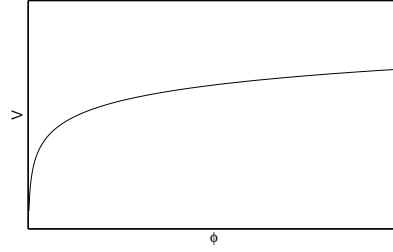


Fig. 5. F - and D -term inflation, colliding brane, mutated hybrid.

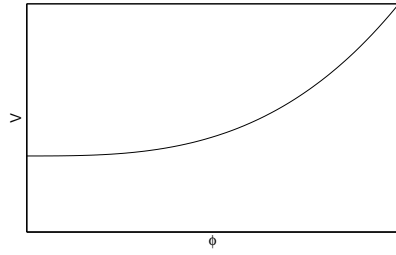


Fig. 6. Tree-level hybrid.

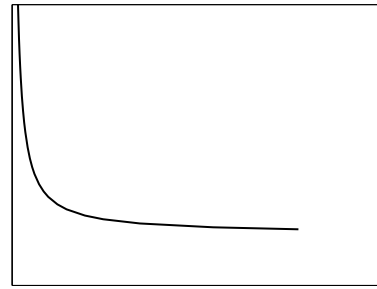


Fig. 7. Dynamical supersymmetry breaking.

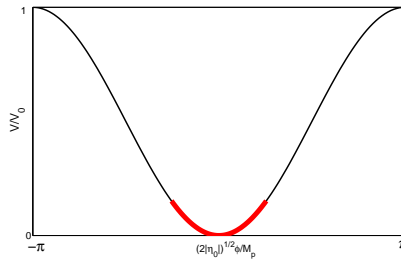


Fig. 8. Natural/chaotic inflation

$$V \simeq V_0 - \frac{1}{4}\lambda\phi^4 + \dots \quad (49)$$

To be precise, the inflaton was supposed to be the GUT Higgs, taken to be practically massless, whose Mexican-Hat potential was generated by a running coupling coming from the non-supersymmetric Coleman–Weinberg potential. The cmb normalization now requires $\lambda = 3 \times 10^{-13}(50/N)^3$. This ruled out the model in its original form, because λ was the GUT gauge coupling with known magnitude of order 10^{-1} . A viable version of the model was obtained [65] by declaring that the inflaton is a gauge singlet, making λ a Yukawa coupling whose value can be chosen at will.

Instead of invoking the approximation (32), we might suppose that the quadratic term dominates while cosmological scales leave the horizon but a higher term dominates soon afterward. The simplest potential of this kind is

$$V = V_0 - \frac{1}{2}m^2\phi^2 - \frac{1}{4}\lambda\phi^4 + \dots \quad (50)$$

A supersymmetric realization of this case making close contact with particle physics is given in [61] (see also [1]), which is very fine-tuned if the inflaton is required to generate the curvature perturbation. There is also a non-supersymmetric realization invoking a Little Higgs mechanism [62, 63], making ϕ a PNCB with a periodic potential. The prediction for this model is the same as for Eq. (30), with the difference that ϕ_{end} will be far below M_{P} making the inflation scale far below 10^{16} GeV.

Turning to hybrid inflation, the simplest possibility is inverted hybrid inflation [64] where the origin remains the fixed point of symmetries, and one simply reverses the sign of m^2 , m_ψ^2 and λ' in the usual hybrid inflation potential (Eq. (51) below). The negative sign of λ' is difficult to arrange especially in a supersymmetric model, and severe fine-tuning is also required [66].

Instead one can make ϕ a PNCB so that it has a periodic potential [67, 62, 63]. The shift symmetry is broken both by the potential $V(\phi)$ and by the coupling of ϕ to the waterfall field. The inflationary trajectory does not pass through the fixed point of the symmetries, and taking the origin to be a maximum of the potential is just an arbitrary choice. Instead of making ϕ a true PNCB, one can arrange that at least it is effectively one during inflation, in the sense that the potential then becomes flat in some well-defined limit [56, 57, 68]. For both types of model it seems possible for the magnitude of the spectrum and the spectral tilt to be in agreement with observation by suitable choice of parameters. The inflation scale can be many orders of magnitude below 10^{15} GeV.

11 Moving toward the origin; power-law potential

In this section we consider potentials of the form illustrated in Figure 5, of either the small-field or medium-field type. We begin with potentials that can be approximated by Eq. (32) with $p < 0$. Such potentials give the prediction (34) for the spectral index and (35) for the tensor fraction.

With $p = -4$, Eq. (32) has been derived in a brane world scenario, where $\mu \sim M_{\text{P}}$ is allowed corresponding to a medium-field model [69]. This is a hybrid inflation model, with the usual potential schematically of the form

$$V(\phi, \chi) = V(\phi) + \frac{1}{2}m^2\phi^2 - \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}\lambda'\chi^2\phi^2 + \frac{1}{4}\lambda\chi^4. \quad (51)$$

At $\phi > \phi_c \equiv m_\chi/\sqrt{\lambda'}$ the waterfall field is driven to zero, leaving $V(\phi)$ given by Eq. (32). The unusually form of $V(\phi)$ here arises because the inflaton field ϕ corresponds to the distance between branes attracted towards each other. Inflation in this model ends when the branes coalesce. Taking the inflaton field to be canonically normalized as usual, the prediction with $\mu = M_P$ is shown in Figure 10.

Colliding brane inflation has the usual η problem, in that the potential is expected to have a term $\frac{1}{2}m^2\phi^2$ with $m^2 \sim H^2$. But the brane world scenario can motivate a non-canonical normalization of a specific form, leading to what is called DBI inflation which can take place even with $m^2 \sim H^2$. We shall not present the results for that case.

At the end of this brane world inflation, F and D strings are typically produced. At present it is not clear how that affects the viability of the model, because the evolution of the string network has not been reliably calculated.

The potential (32) with various values of p had been derived earlier in the context of ordinary field theory, with $\mu \ll M_P$ corresponding to a small-field model. The mechanism, referred to as mutated [70] or smooth [71] hybrid inflation, is the following. The waterfall field is not fixed during inflation, but instead adjusts to continually minimize the potential. The effective potential is then $V(\phi, \xi(\phi))$, and for simplicity the ϕ -dependence at fixed χ is taken to be negligible. In this way [64] one can obtain any $p < 0$ (not necessarily integral) as well as $p > 1$. Taking negative p , the upper bound on r (evaluated by setting $\Delta\phi < M_P$) is shown in Figure 10.

This is a good place to mention another potential of the kind shown in Figure 5;

$$V \simeq V_0 \left[1 - \exp\left(-q \frac{\phi}{M_P}\right) \right], \quad (52)$$

with q of order 1. It occurs if inflation takes place in field space where the kinetic function has a pole, irrespective of the form of the potential [57], with model-dependent values of q such as $q = 1$ or $\sqrt{2}$. It can also be obtained by transforming R^2 gravity or scalar-tensor gravity to the Einstein frame, giving $q = \sqrt{2/3}$. Notice that these modified-gravity theories should not be used in conjunction with the standard supergravity potential, because that potential is evaluated in the Einstein frame.

The potential is supposed to apply in the regime where V_0 dominates, which is $\phi \gtrsim M_P$. Inflation ends at $\phi_{\text{end}} \sim M_P$, and when cosmological scales leave the horizon, we have $\phi \simeq \ln(q^2 N) M_P / q$ and

$$n \simeq 1 + 2\eta = 1 - \frac{2}{N}. \quad (53)$$

The predicted cmb normalization (for $q = 1$ and $N = 50$) is shown in Figure 10 as a cross.

12 F and D term inflation

Now we suppose that the potential is dominated by the loop correction, in a model invoking spontaneously-broken global supersymmetry. We focus initially on the case that the supergravity correction is negligible, asking later whether that is reasonable in specific models. In the regime $\phi \gg \phi_c$ the potential is then given by Eq. (45), while in the limit $\phi \rightarrow \phi_c$ it vanishes (because $M_i(\phi)$ in Eq. (43) vanishes). The mass-squared in Eq. (45) is proportional to some coupling g which controls the strength of

the spontaneous supersymmetry breaking. The potential during inflation is therefore of the form

$$V(\phi) = V_0 \left(1 + \frac{g^2}{8\pi^2} f(\phi) \ln \frac{\phi}{Q} \right), \quad (54)$$

where $f = 1$ for $\phi \gg \phi_c$ and $f \rightarrow 0$ as $\phi \rightarrow \phi_c$. The potential has the form shown in Figure 5.

For $\phi \gg \phi_c$,

$$\eta = -\frac{g^2}{8\pi^2} \frac{M_{\text{P}}^2}{\phi^2} = -\epsilon \frac{M_{\text{P}}}{\phi}. \quad (55)$$

Consider first the regime

$$g^2 \gg 8\pi^2 \phi_c^2 / M_{\text{P}}^2. \quad (56)$$

Slow-roll inflation ends at $\phi_{\text{end}} = 2gM_{\text{P}}^2/4\pi \gg \phi_c$, because $\eta = 1$ there. After *slow-roll* inflation ends, ϕ oscillates about $\langle \phi \rangle = 0$. A few e -folds (of order $\ln(\phi_{\text{end}}/\phi_c)$) of ‘locked’ inflation then occur, until the amplitude falls below ϕ_c .

The integral Eq. (16) is dominated by the limit ϕ giving

$$\phi \simeq \sqrt{\frac{N}{4\pi^2}} g M_{\text{P}}. \quad (57)$$

To be in the desired regime $\phi \ll M_{\text{P}}$ we need $g \ll 1$ which might be in conflict with Eq. (56). Proceeding anyway one finds $n = 1 - 1/N \simeq 0.98$, and the cmb normalization $r = 0.0011(50/N)g^2$. This prediction (with $N = 50$) is shown as a star in Figure 10.

All this is with g in the regime Eq. (56). If we decrease g smoothly to reach the opposite regime $g^2 \ll 8\pi^2 \phi_c^2 / M_{\text{P}}^2$, $\phi(N)$ approaches ϕ_c , the cmb normalization decreases and n approaches 1 [72].

Two versions of this model exist in the literature, referred to generally as F -term [56, 57, 73] and D -term [76, 77] inflation.¹² In both cases, the starting point is a simple global supersymmetry theory with canonical kinetic terms, giving the hybrid inflation potential (51) with $V(\phi)$ perfectly flat.

In the F -term case, g is a Yukawa coupling, which can be chosen to be small yielding a small-field model. The cmb normalization fixes the vev of the waterfall field, as $\Lambda \simeq 6 \times 10^{15}$ GeV. Identifying the waterfall field(s) as a subset of the GUT Higgs fields motivates this value. Turning that around, the GUT model predicts roughly the observed magnitude for the spectrum of the curvature perturbation.

As we are dealing with an F term, the η problem exists; we expect $V \simeq \pm m^2 \phi^2$ with $m^2 \sim H^2$. To have a viable model m^2 needs to be tuned down by a factor of order 0.01 but there is no reason why it should be negligible. The case of positive m^2 has been investigated in [75] and negative m^2 in [3]. The latter case gives an attractive model because it corresponds to hilltop inflation as in Figure (9). After eternal inflation near the hilltop, the field can roll in the negative ϕ direction. After re-defining the origin and reversing the sign of ϕ we recover the small-field model considered in Section 10. Taking the case (56), the spectral index and the height of the potential have been calculated, and are lower than in the original model.

¹² The supergravity potential can be written as the sum of an F term and a D term. With the D term one is driven more or less inevitably to this type of model, but many other possibilities exist with the F term.

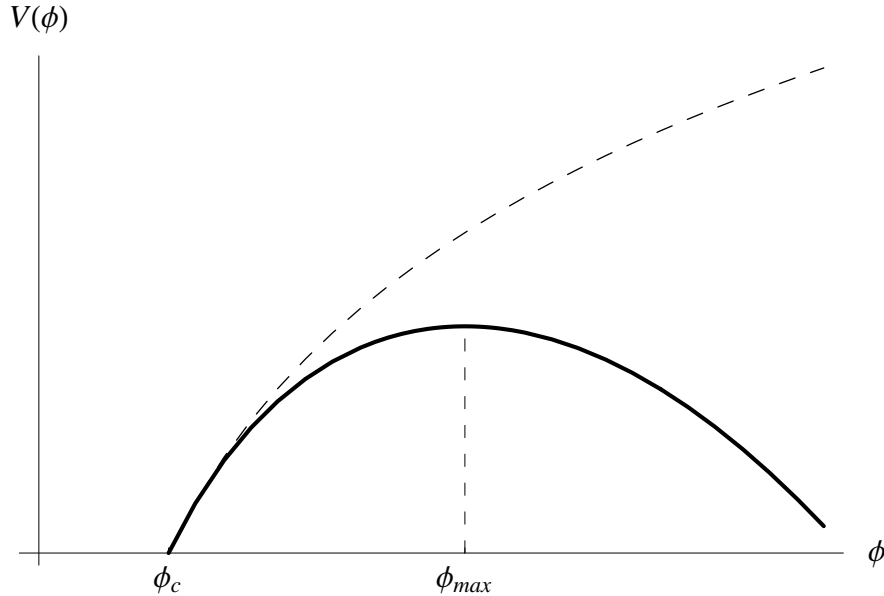


Fig. 9. Sketch of the inflationary potential for the F/D -term scenario in its simplest form, without a tree-level potential (dashed line) and with a concave-downward tree-level potential (continuous line).

In the D -term case, g is a gauge coupling which presumably cannot be small. The vev of the waterfall field has the same cmb normalization as in the F -term case. This vev is expected to be of order the string scale, relating D term inflation directly to string theory.

There is no η problem for the D -term model, but the tree-level potential $V(\phi)$ is still not expected to be flat because we are dealing with a medium-field model where non-renormalizable terms are out of control. There is no particular reason to think that the tree-level $V(\phi)$ will be quadratic, but one may adopt the quadratic form as a parameterization. The case of positive mass-squared was considered in [78, 81], and negative mass-squared in [3, 80]. As in the F term case, it gives an attractive inflation model and with the height of the potential and the spectral index both lower than with the original model.

In both the F and D term models, the inflationary energy scale without a tree-level potential is $V \simeq g^2 \Lambda^4$. Cosmic strings are generically produced with tension $\mu \sim V^{1/2}$, and the cmb constraint $\mu^{1/2} \lesssim 10^{15}$ GeV imposes restrictions on the parameter space.

13 Tree-level hybrid inflation

All of the models considered so far can give a spectral index which is consistent with observation at the time of writing, provided that N is not too far below the expected value ($\simeq 50$). Now we turn to small- and medium-field models which at least

in their simplest form are ruled out by their prediction for the spectral index (as always, on the assumption that the inflaton perturbation generates the curvature perturbation).

Any small- or medium-field model with a concave-upward potential is ruled out. Such models are of the hybrid type, unless the potential becomes concave-downward after cosmological scales leave the horizon. Taking the fixed point as the origin of symmetries, we distinguish between potentials with positive slope as in Figure 6, and with negative slope as in Figure 8.

A negative slope can arise from non-perturbative quantum effects [82]. More usually, one finds models with positive slope as in Figure 6, coming from a tree-level hybrid inflation model with (say) a quadratic potential. The potential including the waterfall field χ is [14] of the form (51) with $V(\phi) = \frac{1}{2}m^2\phi^2$.

A well-motivated tree-level hybrid inflation model, called Supernatural Inflation by its authors [84], uses softly-broken global supersymmetry. The waterfall field is, in our nomenclature, a light modulus [74]. In contrast with most models of inflation, the inflationary scale is low corresponding to $V_0^{1/4} \sim M_S \sim 10^{10}$ GeV, the idea being that there is gravity-mediated supersymmetry breaking both during inflation and in the vacuum, the only difference in the former case being that the last term of Eq. (42) has not yet kicked in. In order to achieve a viable model the masses m_χ and m are taken to be respectively somewhat bigger and smaller than their generic values of order H_* . The observed curvature perturbation is then obtained with λ' just a few orders of magnitude below 1.

The origin $\chi = 0$ is taken by the authors to be, in our nomenclature, a point of enhanced symmetry. The relevant symmetries cannot be those of the Standard Model because $\langle\chi\rangle \sim M_P$. After inflation the waterfall field oscillates about its vev, but it is supposed to decay into SM particles before nucleosynthesis so that it presents no moduli problem. This makes the vev another point of enhanced symmetry, the symmetries now being those of the Standard Model [74].

As with practically all inflation models, the inflaton is invoked just to give inflation and is not part of any extension of the Standard Model that has been proposed for other purposes. Models similar in spirit have been proposed (beginning with [85]) that are based on extensions of the Standard Model that serve other purposes too. They have an even lower inflation scale, corresponding to a mediation strength stronger than gravitational. They invoke fine tunings, which may however be reasonable within the context of string theory and branes. They can give either ordinary or inverted hybrid inflation, but in both cases the spectral tilt is practically zero in contradiction with observation. To avoid this problem though, it seems possible to generate the curvature perturbation during preheating [87].

In considering tree-level hybrid inflation, one has to remember that the coupling of the inflaton to the waterfall field generates a calculable loop correction to the potential, which can be concave-downward and rescue the model. This still leaves a large region of parameter space in which the one-loop correction from this source is negligible [83], though in some part of that space one should still worry about the two-loop correction [84]. In any case the coupling of the inflaton to fields other than the waterfall field can also generate a concave-downward loop correction. We consider this possibility next, in the context of the running-mass model.

A different possibility for generating a concave-downward potential would be to include the leading non-renormalizable term with a negative sign, generating a

maximum as we discussed already for F and D term inflation. The possibility has not been investigated at the time of writing.

14 Running mass models

The loop correction with soft supersymmetry breaking generates a running mass. If the mass belongs to the inflaton we have a running-mass inflation model. The usual model [88] starts with the Supernatural Inflation model that we mentioned earlier. At $\phi = M_P$, the running mass $m^2(\phi)$ is supposed to be of order V_0/M_P^2 , which is the minimum value in a generic supergravity theory. The inflaton is supposed to have couplings (gauge, or maybe Yukawa) that are not too small, and it is supposed that $m^2(\phi)$ passes through zero before it stops running. The running associated with a given loop will stop when ϕ falls below the mass of the particle in the loop.

The potential near $m^2(\phi) = 0$ is flat enough to support inflation. To see this, we can use Eq. (46) which is valid over any small range of ϕ and will therefore be valid around the minimum. It can be written in the form

$$V = V_0 \left[1 + \frac{1}{2} \eta_0 \frac{\phi^2}{M_P^2} \left(\ln \frac{\phi}{\phi_*} - \frac{1}{2} \right) \right], \quad (58)$$

which leads to

$$M_P \frac{V'}{V_0} = \eta_0 \frac{\phi}{M_P} \ln \frac{\phi}{\phi_*}. \quad (59)$$

The potential has a maximum or minimum at $\phi = \phi_*$, at which $\eta = \eta_0$, and near which

$$\eta = \eta_0 \left(1 + \ln \frac{\phi_*}{\phi} \right). \quad (60)$$

A maximum is favoured theoretically, because a minimum requires a hybrid inflation model with ϕ_c tuned to be near the minimum.

To estimate $|\eta_0|$, we can make the crude approximation that Eq. (60) is valid at $\phi \sim M_P$, where $|\eta|$ is supposed to be of order 1. Then

$$|\eta_0| \sim 1 / \ln(M_P/\phi_*). \quad (61)$$

This will give $|\eta_0| \ll 1$ if ϕ_* is exponentially below M_P , and with the reasonable requirement $\phi_* \gtrsim 100 \text{ GeV}$ it gives something like $|\eta_0| \sim 10^{-1}$. For a generic value of $\phi(N)$ this corresponds to $|n - 1| \sim 0.1$ which is outside the observational bound. One can satisfy current observation by choosing the parameters so that $\phi(N) = \phi_*$ around the middle of the cosmological range of scales, corresponding to the spectrum having a maximum at that point [89]. The running of the spectral index at that point is $dn/d \ln k \simeq -2\eta_0^2$, and we are requiring $|\eta_0| \sim 10^{-1}$. This is allowed by present observations with, though it will soon be ruled out or confirmed.

To see whether the condition $\phi(N) \simeq \phi_*$ is reasonable, as well as to calculate the cmb normalization, we need

$$N(\phi) = -\frac{1}{|\eta_0|} \ln \left(\ln \frac{\phi_{\text{end}}}{\phi_*} \ln \frac{\phi_*}{\phi} \right). \quad (62)$$

If slow-roll inflation ends at $|\eta| \sim 1$, and Eq. (60) is still roughly valid there, $|\eta_0| \ln(\phi_*/\phi_{\text{end}}) \sim 1$ and Eq. (62) requires roughly $|\eta_0| \simeq \exp(-N|\eta_0|)$ which is

more or less compatible with $|\eta_0| \sim 0.1$, and also more or less satisfies the cmb normalization with $V_0^{1/4} \sim 10^{-10}$ GeV.

A running mass has also been considered in the context of a two-field modular inflation model [21, 90]. The two real fields are components of a complex field Φ . The maximum of the tree-level potential, chosen as $\Phi = 0$, represents a point of enhanced symmetry, and its height is $V_0^{1/4} \sim 10^{10}$ GeV corresponding to gravity-mediated supersymmetry breaking. Writing $\Phi \equiv |\Phi|e^{i\theta}$, the potential depends on both θ and $|\Phi|$. The tree-level negative mass-squared defined at the origin is supposed to have the generic value corresponding to $|\eta_0| \sim 1$, but interactions cause the mass to run. This turns the maximum into a crater, and it makes the potential very flat at the rim so that inflation can take place there.

There is a family of trajectories characterized by the initial value of θ . The curvature perturbation in this two-field model was calculated from the δN formalism. Near a special value of θ , chosen as zero, θ can be chosen to reproduce the cmb normalization is reproduced with $V_0^{1/4} \sim M_S \sim 10^{10}$ GeV. It seems to be possible to reproduce the observed spectral index by choice of parameters.

15 Large- field models

Now we turn to large-field models. They give a significant tensor perturbation $r \sim 10^{-2}$, which will be observed or ruled out in the near future.

The field variation cannot actually be extremely large, because Eq. (16) requires $\Delta\phi/M_P < \sqrt{2\epsilon_{\max}}N \ll 50$. Two kinds of potential have been considered. One [9] is the Chaotic Inflation potential $V \propto \phi^p$ with p an even integer. The slow-roll parameters are

$$\epsilon = \frac{p^2}{2} \frac{M_P^2}{\phi^2}, \quad \eta = p(p-1) \frac{M_P^2}{\phi^2}. \quad (63)$$

Inflation ends at $\phi_{\text{end}} \simeq pM_P$. When cosmological scales leave the horizon, we find from Eq. (16) that $\phi = \sqrt{2Np} M_P$, giving

$$n-1 = -\frac{2+p}{2N} = -\frac{2+p}{100}, \quad r = \frac{4p}{N} = 0.08p. \quad (64)$$

Current observational constraints practically rule out the case $p \geq 4$. Future observation will rule out or support the remaining case $p = 2$. The cmb normalization for $V = \frac{1}{2}m^2\phi^2$ is $m = 1.8 \times 10^{13}$ GeV, and for $V = \frac{1}{4}\lambda\phi^4$ it is $\lambda = 7 \times 10^{-14}$. If the curvature perturbation is not generated by the inflaton, these become upper bounds, and there is no spectral index constraint.

Another simple possibility is to use a sinusoidal potential

$$V = \frac{1}{2}V_0 \left[1 + \cos \left(\sqrt{2|\eta_0|}\phi/M_P \right) \right]. \quad (65)$$

Here, the origin has been taken to be the maximum of the potential, and $\eta_0 < 0$ is the value of η there. This was called Natural Inflation by its authors [91]. The vev is at $\langle\phi\rangle = -\pi M_P/\sqrt{2|\eta_0|}$.

With this potential $\phi(N)$ is given by

$$\sin \left(\sqrt{\frac{|\eta_0|}{2}} \frac{\phi}{M_P} \right) = \sqrt{\frac{1}{1+|\eta_0|}} e^{-N|\eta_0|}, \quad (66)$$

leading to

$$\epsilon = \frac{1}{2N} \frac{2N|\eta_0|}{e^{2N|\eta_0|} - 1}, \quad \eta = \epsilon - |\eta_0|. \quad (67)$$

The maximum is at $\phi = 0$, and eternal inflation can take place there providing the initial condition for observable inflation. But if $N|\eta_0| \ll 1$, observable inflation itself will not begin until the potential is near the minimum, corresponds to the ‘chaotic inflation’ potential $V = \frac{1}{2}m^2\phi^2$. The prediction in the r - n plane is shown in Figures 3 and 10. We see that the current bound on n requires $r \gtrsim 10^{-2}$. This means that Natural Inflation will eventually be confirmed or ruled out, though it may turn out to be indistinguishable from chaotic inflation.

Large-field models are difficult to understand within the generally accepted rules for constructing field theories beyond the Standard Model, whereby the higher order terms in the expansion (36) are under control only for $\phi \ll M_P$. Some possibilities do exist though.

First, the inflationary trajectory may lie in the space of many fields, corresponding say $\phi = \sum_{i=1}^N a_i \phi_i / \sqrt{\sum a_i^2}$. Then, with say all a_i equal, we can have $\phi \gg M_P$ with each $\phi_i \ll M_P$. This was called Assisted Inflation by its authors [92]. At first sight one might think that the proposal lacks content, since a rotation of the field basis can always make ϕ one of the fields. The point though is that the field theory may select a particular basis, as the one in which the power series (36) is expected to be relevant. It has been argued [93] that this will be the case if each ϕ_i has a sinusoidal potential, leading to what they called N -flation. Then, if inflation takes place near the minimum of the potential one can have ϕ^2 chaotic inflation even though the proportionality $V \propto \phi^2$ does not persist up to the Planck scale.

A second possibility is for the inflationary trajectory may wind many times around the fixed point of the symmetries, at a distance $\lesssim M_P$ from that point. Something like this has been suggested in the context of string theory [94], giving a sinusoidal potential corresponding to Natural Inflation. Finally, it may be possible to evade the general rule that Eq. (36) is out of control at $\phi \gg M_P$, if the field theory is derived from a special higher-dimensional setup. This is the idea of Gauge Inflation [95, 63, 96], where the inflaton is the fifth component of a gauge field living in a 5-d theory, which becomes a PNCB in the 4-d theory. This again can give a sinusoidal potential. None of these proposals allows V to increase continually up to the Planck scale, in the spirit of the Chaotic Inflation proposal.

16 Warm Inflation

In all of the inflation models mentioned so far, energy loss by the inflaton field ϕ is assumed to be negligible on the grounds that ϕ changes only slowly with time. Including this energy loss will give an equation of the form

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + V' = 0, \quad (68)$$

where Γ is some time-dependent quantity. The warm inflation model [97] assumes that Γ is significant, or even dominant ($\Gamma \gg H$).

The extent to which warm inflation is possible was investigated in the GUT hybrid inflation model [98] using an earlier calculation of the energy loss [99]. It does not occur in the original GUT hybrid model but apparently can occur if the inflaton

has a suitable interaction with a spin-half particle. The curvature perturbation in warm inflation receives a contribution from the thermal fluctuation, which dominates the contribution of the vacuum fluctuation if Γ is dominant.

17 Present status and outlook

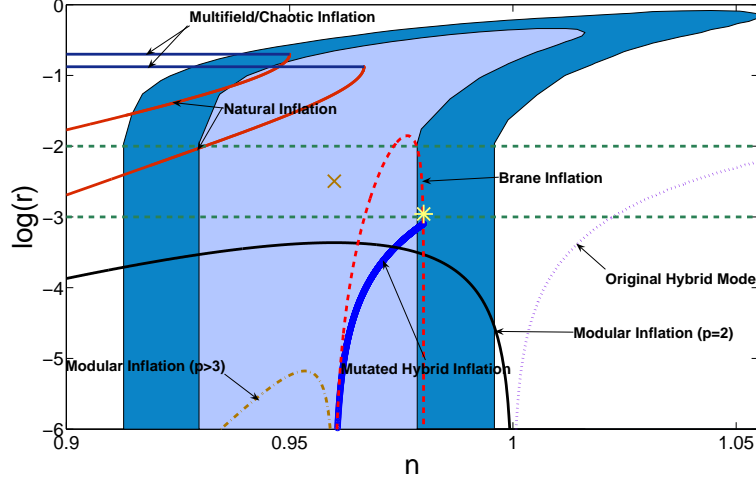


Fig. 10. The shaded regions are the allowed by observation as in Figure 3, and the predictions are described in the text. Planned observation will detect r or give a limit $r < 10^{-2}$, and $r < 10^{-3}$ will probably never be observed.

Figure 10 summarizes most of the predictions that we have been discussing, always assuming that the inflaton perturbation generates the curvature perturbation. (Recall that the alternative was considered in Section 4.5.)

Consider first small- and medium-field models. For these models the tilt is directly related to the curvature of the potential, $n - 1 = 2\eta$. As a result, the recently-observation negative tilt has had a dramatic effect, ruling out whole classes of otherwise attractive models. These include the original tree-level hybrid inflation model, in particular those rather well-motivated versions which invoke during inflation the vacuum supersymmetry-breaking mechanism. The running-mass variant of tree-level hybrid inflation is not yet ruled out, but it will be if the observational bound on the running of n gets much tighter.

Among simple single-field slow-roll models, the ones that agree with observation are modular inflation, and hybrid inflation with a concave-downward potential. The latter can be achieved by what are usually termed simply F - and D -term inflation, involving the loop correction generated by spontaneously broken global supersymmetry. They can also be achieved by mutated hybrid inflation.

All of these simple models give (exactly or as what should be a reasonable approximation) a distinctive prediction for the scale-dependence of the tilt, of the form

$$n - 1 = - \left(\frac{p-1}{p-2} \right) \frac{2}{N(k)}. \quad (69)$$

This gives the scale-dependence (running)

$$\frac{1}{2} \frac{dn}{d \ln k} = - \left(\frac{p-2}{p-1} \right) \left(\frac{n-1}{2} \right)^2. \quad (70)$$

Several years down the line it might be possible to measure this level of running, for instance through a measurement of the 21-cm anisotropy. A confirmation of the above prediction would select within observational uncertainty values for both N and p . If the former were in the relatively narrow range compatible with post-inflationary cosmology, one would probably be convinced that that a model with the relevant p is correct. That would be a truly remarkable development, since it would imply a high inflation scale $V^{1/4} \sim 10^{15}$ GeV and with a sufficiently accurate value of N the reheat temperature would also be determined (assuming continuous radiation domination after inflation).

Now consider the large-field models. The prediction for r and n is compatible with observation for $V \propto \phi^2$, and for Natural Inflation if the period of the potential is not too small. From Figure 3 it is clear that a joint measurement of r and n can rule out these models. Conversely, a measurement of r and n in agreement with one of them would be very suggestive. Again, many years down the line further confirmation could come from a measurement of the running of $n(k)$ and $r(k)$, which goes along the lines indicated in Figure 3. And, again, if such a measurement were compatible with a sensible value for N one would be convinced about the validity of the model, implying again the high inflation scale now $V^{1/4} \sim 10^{16}$ GeV.

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